

DETERMINING THE IMPACT OF REPEATED MEASURES ON POWER FOR VARYING NUMBERS OF INTERMEDIATE MEASUREMENTS

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DOI : <https://www.doi.org/10.56726/IRJMETS43830>

ABSTRACT

In terms of the methodology used in this project, we will determine the impact of the power of formal comparisons between groups, which are based on the final measurement when the intermediate measures are included in the analysis via a linear mixed model, which has been included as a module in ST6034.^[1] This impact of power has been assessed for the various intermediate measurements.

Suppose the assumptions of independence are violated. In that case, we will have the model containing observations from each of the groups, and It would be reasonable to assume freedom within each of these subjects. However, this model can be developed further, and when fitted, adjustments are made to account for the lack of independence. The methodology is known as Repeated Measures ANOVA.^[2]

Even with these adjustments made to the subjects, these Repeated Measures ANOVA models don't impose compound symmetry on the correlation within these subjects. We will know the relationship and the different factors that affect power. We will also be understanding the interrelationship between the statistical significance, power, and effect size. We will look at the values which determine the power with relation to the other variables. We will look at the mean and standard deviation of the sample size with the other variables known. The learning process with the hypothesis, sample subjects, the sample distributions, the calculation of the hypothesis tests, the confidence intervals, and the effect size determine the power. We will also establish relationships with the relational studies with the statistics to analyze the data with the correlational model or the experimental designs.

Keywords: Linear Mixed Models, Repeated Measure Analysis, GLS, REML.

I. INTRODUCTION

The Regular p-value calculations in the repeated measures anova are accurate if the distribution of the variables has compound symmetry. This means that all the response variables have the same variance and each pair of response variables share a common correlation.^[3]

$$\sigma^2 \begin{bmatrix} 1.0 & \rho & \rho & \rho \\ & 1.0 & \rho & \rho \\ & & 1.0 & \rho \\ & & & 1.0 \end{bmatrix} = \begin{bmatrix} \sigma_b^2 + \sigma_e^2 & \sigma_b^2 & \sigma_b^2 & \sigma_b^2 \\ & \sigma_b^2 + \sigma_e^2 & \sigma_b^2 & \sigma_b^2 \\ & & \sigma_b^2 + \sigma_e^2 & \sigma_b^2 \\ & & & \sigma_b^2 + \sigma_e^2 \end{bmatrix}$$

Compound Symmetry is the simplest covariance structure that includes correlated errors within the subject. Compound symmetry assumes that the correlation between any pair of values of a given subject is the same regardless of how close together or far apart they are in time. There is another class of models which allow much greater flexibility in what structure to apply/assume for the correlations and hence the covariance matrix within subjects are called as Linear Mixed Models.

Linear Mixed Models are sometimes called as Multilevel models or Hierarchical models depending upon the type of regression models. Linear Mixed Models explain two kinds of effects which are fixed effects and the other one is random effects. Fixed Effects is the variation that is explained by the independent variables of interest and Random Effects is the variation that is not explained by the independent variable of interest.^[2]

Since Linear Mixed Models includes the mixture of fixed and random effects, it's called as a mixed model. The Random effects gives the structure to the ϵ (Error Term).

$$Y = Xb + Zg + e$$

Y is a vector of n observations. (note independence is not specified).

X is the design ($n \times p$) matrix (fixed effects).

β is a vector of p unknown model parameters.

γ is a vector of q unobservable random effects.

Z is the design ($n \times q$) matrix (random effects).

ϵ is a vector of error terms.

In terms of methodology used in this project, we will determine the impact of the power of formal comparisons between groups which are based on the final measurement when the intermediate measurements are included in the analysis via a linear mixed model which has been included as a module in ST6034.^[1] This impact of the power has been assessed for the various intermediate measurements.

If the assumptions of the independence are violated, we will have the model containing observations from each of the groups and It would be reasonable to assume independence within each of these subjects. However, this model can be developed further and when fitted, adjustments are made to account for the lack of independence. The methodology is known as Repeated Measures ANOVA.^[2]

Even with these adjustments made to the subjects, these Repeated Measures ANOVA models doesn't impose compound symmetry on the correlation within these subjects.

II. METHODOLOGY

Comparison Of Linear Mixed Model In Sas And R

An orthodontic study was conducted on 27 children, 11 girls and 16 boys, all of whom were age eight at the beginning of the course. On each child, the distance from the centre of the pituitary to the pterygomaxillary fissure was measured (in mm) every two years through age 14. The study objectives were to determine if the distances were larger for boys than for girls and if the rate of change in the outcome differed between boys and girls.

SAS: Orthodont.dat

R: Orthodont {nlme}

The Orthodont data frame has 108 rows and four columns of the change in an orthodontic measurement over time for several young subjects.^[1]

AIC & BIC from the log-likelihood (LL) – SAS Calculation

The AIC is defined as the log-likelihood term penalized by the number of model parameters. The larger the likelihood, the better the model. The more parameters, the worse the model.

Because the likelihood is often calculated as negative-two-log-likelihood, this formulation is usually found: $AIC = -2LL + 2d$, with $-2LL$ being the negative-two- loglikelihood and d the dimension of the model. Generally, smaller numbers of AIC are better than larger numbers.^[1]

$BIC = -2LL + d \log(n)$, Here LL denotes the maximum value of the (possibly restricted) log-likelihood, d the dimension of the model, and n the number of observations.

AIC & BIC from the log-likelihood – R Calculation

This generic function calculates the Bayesian information criterion, also known as Schwarz's Bayesian criterion (SBC), for one or several fitted model objects for which a log-likelihood value can be obtained, according to the formula $-2\log\text{-likelihood} + n\text{par} \log(\text{nobs})$, where $n\text{par}$ represents the number of parameters and nobs the number of observations in the fitted model.^[1]

This generic function calculates the Akaike information criterion for one or several fitted model objects for which a log-likelihood value can be obtained, according to the formula $-2\log\text{-likelihood} + 2 \text{ npar}$, where npar represents the number of parameters in the fitted model. When comparing fitted objects, the smaller the AIC, the better the fit.

The one-way analysis of variance (ANOVA), also known as one-factor ANOVA, is an extension of independent two-samples t-test for comparing means in a situation where there are more than two groups. In one-way ANOVA, the data is organized into several groups based on one single grouping variable (also called factor

variable). This tutorial describes the basic principle of the one-way ANOVA test and provides practical ANOVA test examples in R software.^[1]

Repeated Measures Analysis Using Baseline Simulation Study

Dependent variables are estimated using the baseline simulation models in the absence of the hypothesized causal effects. The Multivariate normalization used in this section is commonly used in the sequential regression analysis. Baseline simulations are developed to capture the critical patterns in the data which are independent of the hypothesized effects. Baseline simulation models compare the ways with the designs which are formed using the explanatory models [i.e. Linear Model]. These insights are derived to improve the explanatory models. Baseline simulation is carried out in the next section ^[2] using the modified baseline simulation study.

Baseline simulation modelling has received increasing attention through the social and biological sciences experiments. Baseline simulation study raises more methodologies which are general and are related to the scientific inquiries.

In this section, we are having a comparison between an explanatory model [Linear Model with T3 Only] and a baseline model [Power estimated using the contrast of T3]. However, the baseline models have been developed using the data simulated from the $mvrnorm^{[12]}$ for Group A (n=20) and Group B (n=20) for 5000 Simulations. Initially, the number of simulations was set to 1000 simulations. Still, the character of the baseline models changed a bit over time as we have developed more sophistication ^[2]. As we have migrated towards the different fields, we have increased the number of simulations to 5000 simulations.

Although the explanatory model has a more complex alternative, using the P values generated from the models, we can have an estimated power. The Statistical Power of a simulation study is the conditional probability given on a dependent effect size where the hypothesis test will reject the null hypothesis correctly. Using this power analysis, we can conclude that the effect is there when there is one. The situation underscores the power analysis that a particular study can be replicated precisely over and over again ^[2]. For example, if the Power of the course is 65%, Which means that 65% of the replication of the study will appropriately reject the null hypothesis.

To estimate the Power, we can simulate data replications of the study and conduct repeated hypothesis test and derive a percentage from the studied phenomena. In our research, we are using 5000 Simulations of Power to exhibit statistical independence, which changes randomly over time. We can estimate the Power using simulation by replicating the study and performing the hypothesis tests. The Proportion of trials where the null hypothesis is rejected is the estimated Power.

Each of the replications is based on the same set of normalized data (i.e. has the same effect size, same sample size, the same level of individual variation and the same level of group variation) ^[3].

The Idea of the baseline simulation model is used in a sequence of regression calculations. The series of the baseline models include both the control variables and other variables. We are running our 5000 simulations on a model which is randomized having Group A and Group B with time points T1, T2, T3.

Table 1: Baseline Simulation Values for the simulation study

Group	T1	T2	T3
A (n = 20)	$N(\mu=100, \sigma=5)$	$N(103, 5)$	$N(102, 5)$
B (n = 20)	$N(100, 5)$	$N(105, 5)$	$N(110, 5)$

The baseline models' simulations in these regressions differ in significant ways of having 0.25, 0.5, 0.75 autoregressive structures. The appropriate repeated measures analysis was studied, and the first power calculation of the sequence is given in the results. The simulated data is assumed to have complete independence. REML in R is used for the repeated measures, and the baseline model's Power was calculated for 1000, 2000, 3000 simulations.

The GLs method used in this Restricted Maximum Likelihood estimator. The REML Method gathers estimators which are not obtained from the whole likelihood function but are the part of the which is associated with the fixed effects of the linear model. In other words, if $y = Xb + Zu + e$, where Xb is the Analyzing Linear Models With

Proc MIXED fixed effects part, Zu is the random effects part and e is the error term, then the REML estimates are obtained by maximizing the likelihood function of $K'y$, where K is a full rank matrix with columns orthogonal to the columns of the X matrix, that is, $K'X = 0$. It leads to REML estimator of the variance-covariance matrix of y, say V. It does not depend on the choice of matrix K. [3]

REML estimators are unbiased, and they do not have to be equal to those obtained from PROC GLM. Power is the frequency finding a significant result when there is an actual effect, as explained above. The Idea of the power simulation is to simulate the study lots of times and count the number of times the result is significant. The Power simulation can be varied using a different number of parameters, different number of assumptions and calculate the Power accordingly. The only downside of the power analysis is flexible and can be challenging to set up the right comparisons. We have come up with a series of assumptions and comparisons which we will see in the next section.

III. MODELING AND ANALYSIS

DIFFERENCE IN NUMBERS OF REPEATED MEASUREMENTS ANALYSIS

Repeated measures refer to the observational studies or the experimental designs where the subject is measured at several points with a change in time. This is also called as Longitudinal data. During this simulation study, we have already gone through Subjects A and B having 20 data points and Three-time intervals. Now we are extending the survey to Five-time intervals and Eight-time intervals.

As the Simulation Study, we use `mvrnorm[4]`. We have a typical design where experimental units are allocated randomly to time points. We have a series of time intervals. We have 2 Groups A and B having 20 data points each.

In this example design, we have observed 40 points with 3-time series. Determining the Power using contrast at T5 and comparing it with the Power at T5 estimated using the linear model will be the focus of our analysis.

Table 2: T5: Gradually increasing difference between groups

1.	Group	T1	T2	T3	T4	T5
	A (n = 20)	$N(\mu=100, \sigma=5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$
	B (n = 20)	$N(100, 5)$	$N(102, 5)$	$N(104, 5)$	$N(106, 5)$	$N(108, 5)$

We have assessed the time rationale for the inclusion of the repeated measures modelling design element by determining the Power of the simulation provided with the Gradually Increasing difference between the groups with the Power calculated using T5 only analysis. The repeated measures statistical analysis includes the "repeated measures ANOVA", "Repeated measures analysis of variance" and "the mixed model with the subject having the random effect".[4]

Simulation is used for estimating power or sample size requirements when the study is complex. The Sample Size and the Power Estimations depends on the variance estimates of the fixed effects on the mixed model.

$$\text{VAR}(\hat{\beta}) = \left(\sum_{i=1}^N X_i' V_i^{-1} X_i \right)^{-1}$$

Where V_i is called the Y_i 's unconditional variance. The probability of the marginal average and conditional mean is not equal. While the estimation approaches are different, the results are the same as with the ANOVA. However, now we are using a tool that can handle additional time points, continuous covariates with possibly nonlinear relationships, different types of outcome variables, other correlational structures among the observations, etc.[11], if one looks at the help file for `gls`, one will note that it suggests using `lmer` for unbalanced designs and other situations (see the appendix for code).[5]

Table 3: T8: Gradually increasing difference that levels out

Group	T1	T2	T3	T4	T5	T6	T7	T8
A (n = 20)	$N(\mu=100, \sigma=5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$
B (n = 20)	$N(100, 5)$	$N(102, 5)$	$N(104, 5)$	$N(106, 5)$	$N(108, 5)$	$N(108, 5)$	$N(108, 5)$	$N(108, 5)$

CONSIDERING LOWER VALUES OF μ_B (MEAN)

A slightly more complicate baseline simulation model for the time series data assumes that the variables continue to change frequently. Here the Data Consists of five-time series and eight-time series with the power analysis produced following T5 and T8. We are calculating with Autoregressive structure at 0.25, 0.50, 0.75 with 5000 simulations. By one measure, the baseline model produces more accurate results with Group A and Group B having 20 data points each.

Table 4: T5: Gradually increasing difference between groups

Gradual Increasing difference between groups	Values of μ_B
$\mu_B = 101$	100,102,104,106,108
$\mu_B = 102$	100,101.75,103.5,105.25,107
$\mu_B = 103$	100,101.5,103,104.5,106
$\mu_B = 104$	100,101.25,102.5,103.75,105
$\mu_B = 105$	100,101,102,103,104
$\mu_B = 106$	100,100.75,101.5,102.25,103
$\mu_B = 107$	100,100.5,101,101.5,102
$\mu_B = 108$	100,100.25,100.5,100.75,101

Considering lower values of μ_B at T5 (107, 106, 105, 104, 103, 102, 101) and Using the same relative gradual increases from T1 to T5 (e.g. μ_B at T5 = 104. Then T2 = 101, T3 = 102, T4 = 103, T5 = 104). Group A fixed, as above. These changes are only applied to Group B. In general, there are no basic changes in the group A of the simulation model where we have evaluated the inertial tendencies associated with the dependent variable, and we can focus on the investigation of the proposed effects on the time-to-time changes which are either percentage changes or absolute changes.[6]

Table 5: T8: Gradually increasing difference that levels out

Gradual Increasing difference between groups that levels out	Values of μ_B
$\mu_B = 101$	100,102,104,106,108,108,108,108
$\mu_B = 102$	100,101.75,103.5,105.25,107,107,107,107
$\mu_B = 103$	100,101.5,103,104.5,106,106,106,106
$\mu_B = 104$	100,101.25,102.5,103.75,105,105,105,105
$\mu_B = 105$	100,101,102,103,104,104,104,104
$\mu_B = 106$	100,100.75,101.5,102.25,103,103,103,103
$\mu_B = 107$	100,100.5,101,101.5,102,102,102,102
$\mu_B = 108$	100,100.25,100.5,100.75,101,101,101,101

The explicit estimation uses the baseline models, which provides the potentially valuable information which is used for the interpretation of the effects which are observed. This is one of the reasons why we have tended to implement with the other dependent variables which are used for getting the power analysis from changes which occur period to period. The next section focuses on the distribution of the repeated measure using the higher values of σ (Standard Deviation). When we use the baseline simulation models to capture the trends in the simulation study of T5 and T8, we can identify and evaluate accelerations or decelerations in the power analysis according to changes which were made.

CONSIDER HIGHER VALUES OF $\sigma_A = \sigma_B$

We propose a simulation-based methodology for estimating the power values if the baseline simulation study is tweaked with higher amounts of σ_A and σ_B . We will also be using the same baseline simulation study to

compare the power values with higher values of σ_A and having a constant σ_B . We surveyed for the T5 and T8 analysis, and we will be looking at why we did this study in this section.

The Datasets in the simulation study has the likelihood function which is constructed using the Autoregressive process having values 0.25, 0.50, 0.75. The simulation approach consists of the mean and the standard deviation from the samples, which uses a set of accepted parameter values. We repeat the procedure for all the stages of parameter values which are accepted.[7]

To reject the null hypothesis and have the power simulation, we look for a big enough difference between the subjects and the groups. We assessed the results with 40 studies which used Repeated measures analysis and the time point 5, which doesn't use RMA, and we can investigate the reports evaluate the normality of the residual errors. Concerning reporting, we have determined the group's data with the homogeneity of the variance for at least one of the outcomes which was designated with the standard deviation values.[8]

Many studies have made a series of increasingly complex calculations which applies the baseline simulation model methodology in a sequence. We can also evaluate the same investigation with the observed data, which has eight-time points where each group have 20 data points and creating a 5000 simulation. As many of the power analysis has multiple outcomes, we can use the power generated from RMA Analysis compared with the non-RMA Analysis.

DIFFERING INCORRECTLY SPECIFIED CORRELATION STRUCTURES

In this section, we will be looking at the criteria of the variable selection in the function modelling of the mean and the correlation structure selection of the potential candidates in the variance-covariance modelling. The baseline study is carried out with AR(1) correlation structures; Other correlation structures are mentioned below[9].

The Generalized estimation of the correlation structures is used for this kind of baseline simulation we use the longitudinal data analysis where it can be accounted for the cluster within the correlations without working with the variance-covariance structure. The estimation of the variance is consistent for the right variance matrix[13] which has the estimated parameters even when the assumed variance correlation structure is specified incorrectly.

First Order Autoregressive Structure

This is the simplest autoregressive model which uses the most recent outcome of the baseline models time, and that is in turn used to predict the future values. For a time series Y_t such a model is called a first-order autoregressive model, often abbreviated AR(1), where the one indicates that the order of autoregression is one[10]

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + u_t$$

is the AR(1) population model of a time series Y_t

AR(1)

$$\begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 \\ \rho & 1 & \rho & \rho^2 \\ \rho^2 & \rho & 1 & \rho \\ \rho^3 & \rho^2 & \rho & 1 \end{bmatrix}$$

Other Covariance Structures that might be appropriate A covariance matrix with a diagonal structure. The diagonal covariance matrix is known as the variance components model. The covariance matrix that contains specified variances along the diagonal. [10]

CS				
6	1	1	1	1
1	6	1	1	1
1	1	6	1	1
1	1	1	6	1

A covariance matrix with compound symmetry. The compound symmetry model is the sum of a constant matrix and a diagonal matrix. This structure forms a covariance matrix provided that the diagonal elements are large relative to the off-diagonal elements. [10]

A covariance matrix with Toeplitz structure. A Toeplitz matrix has a banded structure. The diagonals that are parallel to the main diagonal are constant. If the diagonal elements are large relative to the off-diagonal elements, then the Toeplitz matrix is positive definite.[10]

toep			
4	1	2	3
1	4	1	2
2	1	4	1
3	2	1	4

A covariance matrix with a first-order autoregressive (AR1) structure

A first-order autoregressive (AR(1)) structure is a Toeplitz matrix with additional structure. Whereas an $n \times n$ Toeplitz matrix has n parameters, an AR(1) system has two parameters. The values along each diagonal are related to each other by a multiplicative factor.[10]

ar1			
1	0.25	0.0625	0.015625
0.25	1	0.25	0.0625
0.0625	0.25	1	0.25
0.015625	0.0625	0.25	1

This one includes the independence estimating the power analysis using different variance correlation structures. These independent estimated of the model vector which should be processed into a single estimate of the particular vector.[11]

DIFFERENCING VARIABILITY IN GROUPS

The standard approach of the models is to estimate the frequency response of the measure. We will now take a look at using different. We will be using increasing variability in one group with the variability being varying from 5,6,7,8,9. We will be doing this analysis in only one group. This analysis will give us a reasonable definition of the modal vector that the coefficient will be able to measure the matrix, which is estimated to detect such potential problems in the variability.

Table 6: T5: Gradually increasing difference between groups having σ_B increasing

Group	T1	T2	T3	T4	T5
A (n = 20)	$N(\mu=100, \sigma=5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$
B (n = 20)	$N(100, 5)$	$N(102, 6)$	$N(104, 7)$	$N(106, 8)$	$N(108, 9)$

The Datasets in the simulation study has the likelihood function which is constructed using the Autoregressive process having values 0.25,0.50, 0.75. The simulation approach consists of the mean and the standard deviation from the samples, which uses a set of accepted parameter values.[From The previous Section] We repeat the procedure for all the settings of parameter values which are accepted.[12]

Table 7: T8: Gradually increasing difference between groups which levels out having σ_B increasing

Group	T1	T2	T3	T4	T5	T6	T7	T8
A (n = 20)	$N(\mu=100, \sigma=5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$	$N(100, 5)$
B (n = 20)	$N(100, 5)$	$N(102, 6)$	$N(104, 7)$	$N(106, 8)$	$N(108, 9)$	$N(108, 9)$	$N(108, 9)$	$N(108, 9)$

The Power of the test is usually obtained with the associated non-central distribution[12]. Several hypothesis tests can be tested, but there are two most common hypothesis tests. The power analysis that one does not reject the null hypothesis when it is false. The Power of the test which is calculated as 1-beta and therefore the GLS function can be able to compute the Power when the other parameters for the variance are given.

Repeated measures analysis needs the ANOVA for the cases in the one observation to be directly linked with the circumstances in all the other statements which we have the variance-covariance structure. Considering the baseline differences that might affect the outcome could be the main parameters[13]. The Repeated measures ANOVA design is appropriate for within the subject matters. This can often result in repeated measurements, which has actual effects between the subjects ANOVA.

IV. RESULTS AND DISCUSSION

This function fits a linear model using generalized least squares. The errors are allowed to be correlated and have unequal variances. Gls is a slightly enhanced version of the Pinheiro and Bates gls function in the nlme package to make it easy to use with the rms package and to implement cluster bootstrapping (primarily for nonparametric estimates of the variance-covariance matrix of the parameter estimates and for nonparametric confidence limits of correlation parameters).

Comparison of CDF's

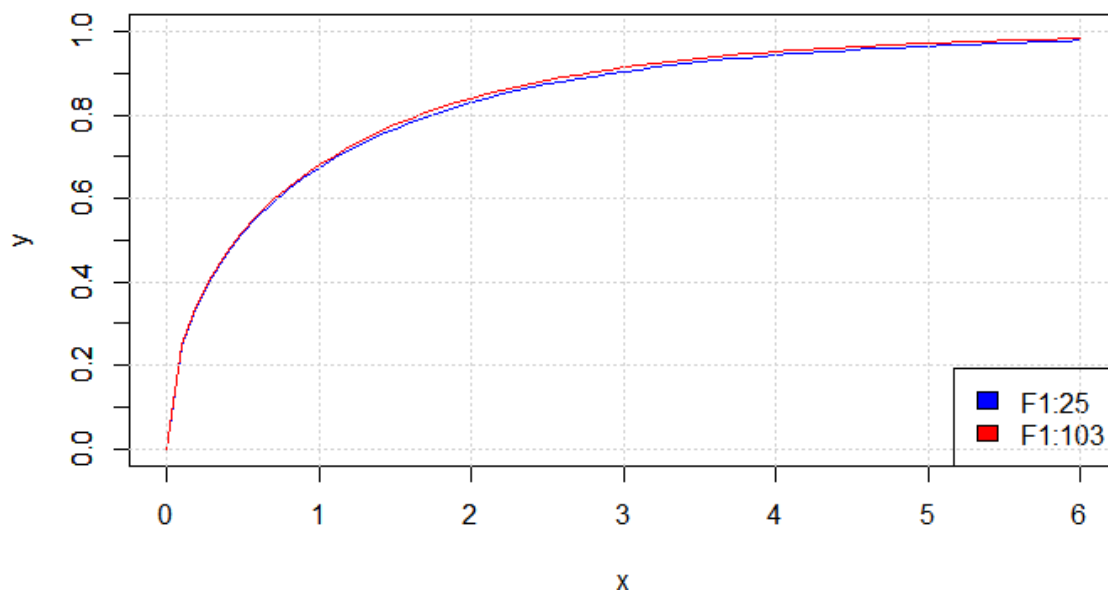


Figure 1: Graph which shows comparison of CDF's

Table 8: Results of AIC, BIC and Loglikelihood Values

	AIC	BIC	LogLik
SAS	454.6	474.1	424.6
R	454.6432	494.8752	(-2)(-212.3216)

RESULTS OF REPEATED MEASURES ANALYSIS USING BASELINE SIMULATION STUDY

Although the explanatory model has a more complex alternative, using the P values generated from the models, we can have an estimated power. The Statistical Power of a simulation study is the conditional probability given on a dependent effect size where the hypothesis test will reject the null hypothesis correctly. Using this power analysis, we can conclude that the effect is there when there is one. The situation underscores the power analysis that a particular study can be replicated precisely over and over again[2]. For example, if the Power of the course is 65%, Which means that 65% of the replication of the study will appropriately reject the null hypothesis.

Table 9: Results of T3, AR(1) having 5000 simulations.

T3 Analysis, AR(1), 5000 Simulations	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
	86.6667	98.9667	78.7667	98.8	74.677	97.8

As the baseline model offers a single coherent explanation for the number of simulations 5000 simulations fitted our needs. The Power was estimated for the overall comparison between Group A and Group B, and the Power 1 was calculated from deriving the contrast for only T3 using repeated measure analysis gls function. Power 2 was calculated considering only T3 in the model using an appropriate linear model analysis. The Regression coefficients are unreliable indicators of the importance of independent variables[3], and the level of significance is associated with the estimates of the statistical value.

RESULTS OF DIFFERENCE IN NUMBERS OF REPEATED MEASUREMENTS ANALYSIS

During the analysis, the number of simulations was administered as 5000, As there weren't much of a difference in Power between 1000, 2000 and 3000 simulations. Repeated measures correlation is the method which is used for determining the association within multiple subjects. Repeated observations were modelled using multivariate likelihood function[6]

Table 10: Results of T5 Analysis, Gradually Increasing difference between groups

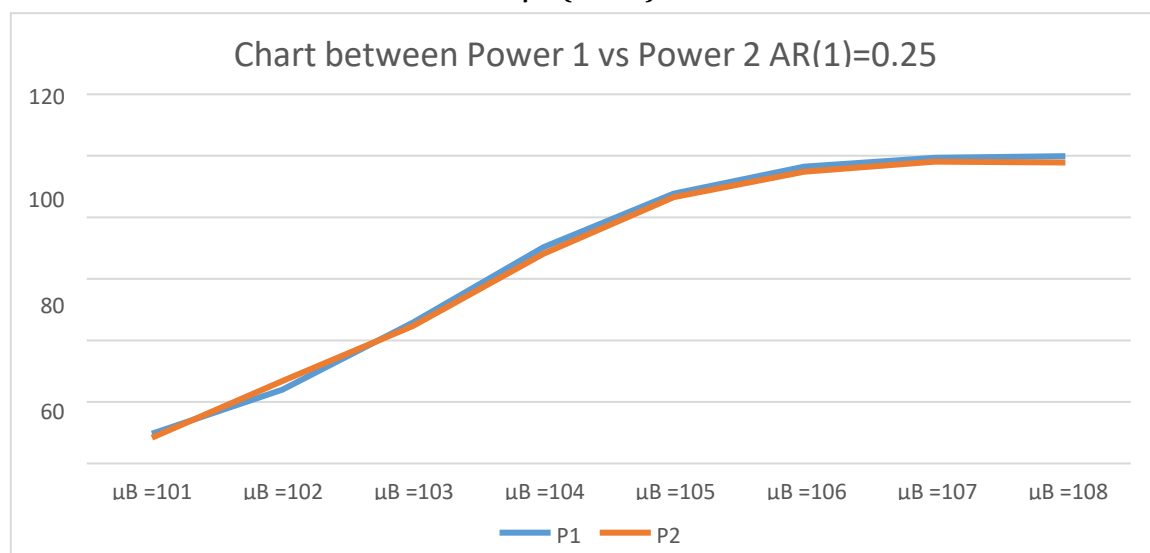
Gradual Increasing difference between groups	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
	99.88	99.84	99.88	99.8	99.92	99.9

This kind of process model is called the exponential decay model where the models grow rapidly and then levels off to become asymptotic to the upper limit[6]. Many models for the longitudinal theories start by capturing the assumptions about the period to period change. They have the inertia about the magnitude and the rates of changes. This will allow the baseline models, which assumes that the dependent variable will not change to fit the time series data over time very well.

Table 11: Results of T8 Analysis, Gradually Increasing difference between groups that levels out

Gradual Increasing difference between groups that levels out	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
	99.92	99.82	99.90	99.82	99.94	99.88

RESULTS OF CONSIDERING LOWER VALUES OF μ_B (MEAN)



The focus in the next section will be to explicitly simulate random effects and try to interpret the complex differences in the trajectories using higher values of standard deviation. We did two kinds of simulation wherein one simulation uses $\sigma A = \sigma B$, and the other simulation has only changes in σA with constant σB .

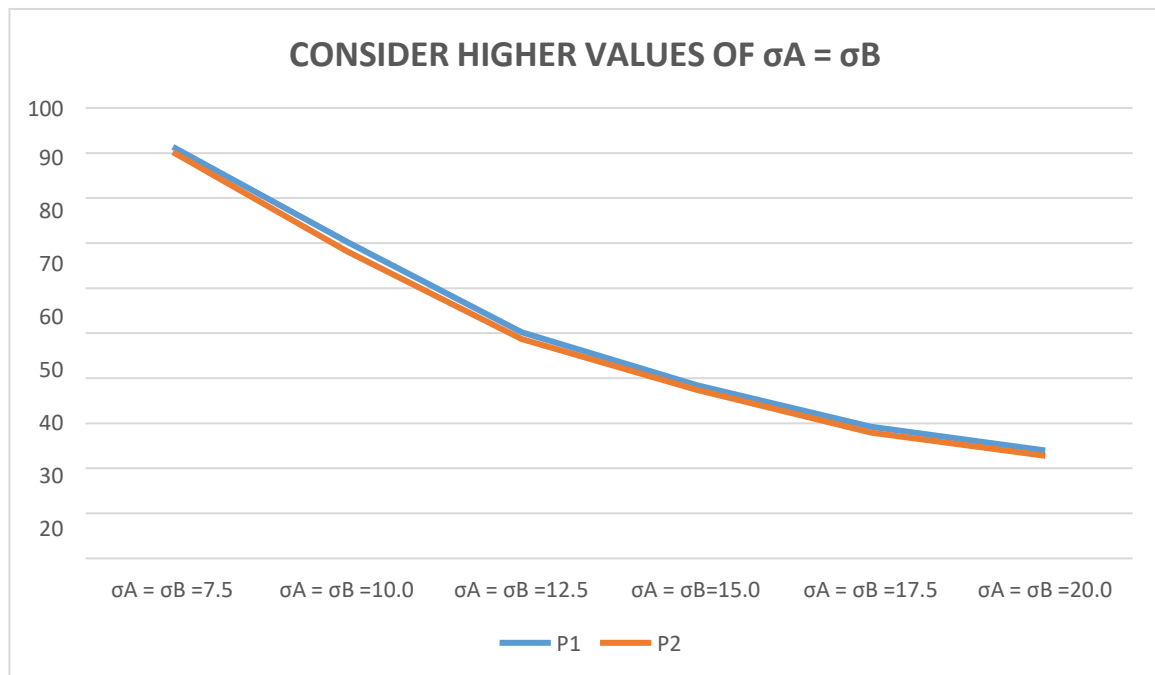
Table 12: Results of T5 Analysis, Gradually Increasing difference between groups

Gradual Increasing difference between groups	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
$\mu B = 101$	9.62	8.34	8.62	9.34	9.4	9.34
$\mu B = 102$	23.92	26.78	23.96	21.47	23.92	22.85
$\mu B = 103$	45.78	44.68	45.77	44.58	45.8	44.33
$\mu B = 104$	70.22	68.14	70.22	66.18	70.28	68.155
$\mu B = 105$	87.68	86.58	87.5	86.68	87.7	86.54
$\mu B = 106$	96.42	94.86	96.3	95.7	96.75	95.45
$\mu B = 107$	99.38	98.1	99.8	99.1	99.74	99.1
$\mu B = 108$	99.88	97.84	99.7	99.3	99.65	99.84

Table 13: Results of T8 Analysis, Gradually Increasing difference between groups and levels out

Gradual Increasing difference between groups that levels out	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
$\mu B = 101$	7.32	9.36	8.2	9.8	7.32	9.36
$\mu B = 102$	11.24	23.46	13.2	21.98	11.24	23.46
$\mu B = 130$	19.28	45.12	19	44.9	19.28	45.12
$\mu B = 104$	30.4	69.54	38.2	65.18	30.4	69.54
$\mu B = 105$	44.88	86.34	52.06	89.18	44.88	86.34
$\mu B = 106$	58.64	95.94	54.66	94.9	58.64	95.94
$\mu B = 107$	71.94	99.12	72.3	21.98	71.94	99.12
$\mu B = 108$	81.54	99.82	80.8	99.8	81.54	99.82

The greater the sample size, the higher than the power. This is the principle which is used in the previous graph. We also learned that the preceding section that anything which increases the value of the denominator, the power decreases. Anything that decreases the ability, where the denominator of the test statistic, larger the sample size, we will have more data to analyze.

RESULTS OF CONSIDER HIGHER VALUES OF $\sigma A = \sigma B$


The means and the standard deviation have been altered in the last section as well as this section where we have the standard deviation, which keeps increasing for both groups. In the next study, we will be creating a comparison where the standard deviation changes for only one group.

Table 14: Results of T5 Analysis, Gradually Increasing difference between groups

Gradual Increasing difference between groups	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
$\sigma A = \sigma B = 5.0$	99.88	99.84	99.88	99.8	99.92	99.9
$\sigma A = \sigma B = 7.5$	91.34	90.2	91.4	90.38	91.58	90.66
$\sigma A = \sigma B = 10.0$	70.22	68.18	70.88	69.38	70.68	68.88
$\sigma A = \sigma B = 12.5$	50.14	48.7	52.06	50.18	52.4	50.16
$\sigma A = \sigma B = 15.0$	38.44	37.42	38.2	36.56	38.7	37.02
$\sigma A = \sigma B = 17.5$	29.22	27.92	29	27.9	29.92	28.46
$\sigma A = \sigma B = 20.0$	23.92	22.76	23.2	21.98	23.76	22.58

This directionality refers to the variance change in power analysis—the Non Directional hypothesis where the experimental group has higher power than the usual subjects. We can see that as the variability increases the power decreases. Our sample data have more significant variability. Random sampling error is more likely to produce considerable differences between the experimental groups even when there is no real effect. If the sample data in Study A have sufficient variability, a random error might be responsible for the massive difference.

Table 15: Results of T5 Analysis, Gradually Increasing difference between groups

Gradual Increasing difference between groups	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
$\sigma B = 5.0$	99.88	99.84	99.88	99.8	99.92	99.9
$\sigma B = 7.5$	97.42	96.98	97.5	96.8	97.42	97.12
$\sigma B = 10.0$	88.72	87.52	88.62	87.16	88.44	87.04

$\sigma_B = 12.5$	74.58	73.4	74.98	73.22	74.66	73.46
$\sigma_B = 15.0$	61.4	59.42	61.18	59.2	61.72	60.16
$\sigma_B = 17.5$	49.42	47.64	49.28	49.06	50.18	48.54
$\sigma_B = 20.0$	40.36	39.46	40.62	40.04	40.74	39.92

Table 16: Results of T8 Analysis, Gradually Increasing difference between groups and levels out

Gradual Increasing difference between groups that levels out	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
$\sigma_A = \sigma_B = 5.0$	99.9	99.82	99.86	99.82	99.94	99.88
$\sigma_A = \sigma_B = 7.5$	92.98	91.26	91.96	90.64	92.18	91.16
$\sigma_A = \sigma_B = 10.0$	72.14	70.32	71.96	68.78	71.58	69.44
$\sigma_A = \sigma_B = 12.5$	52.24	51.32	53.54	50.1	52.84	51.24
$\sigma_A = \sigma_B = 15.0$	38.38	38.44	39.2	39.2	39.32	37.96
$\sigma_A = \sigma_B = 17.5$	29.44	30.32	29.52	29.36	30.08	29.34
$\sigma_A = \sigma_B = 20.0$	23.08	23.92	24.48	22.96	23.88	24.02

To reject the null hypothesis, we look at the big difference between the groups. We have two kinds of errors which is Type I error and Type II error. Experimental studies need to correctly detect when there is a real difference in the groups and also when there is no significant difference in the groups, which are the error. When the difference between the subjects is not substantial enough to reject the null hypothesis, we either retain the null hypothesis correctly, or we have a Type I error.[12]

Table 17: Results of T8 Analysis, Gradually Increasing difference between groups and levels out

Gradual Increasing difference between groups that levels out	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
$\sigma_B = 5.0$	99.68	99.84	99.88	99.8	99.75	99.9
$\sigma_B = 7.5$	97.8	96.98	97.5	96.8	97.9	97.56
$\sigma_B = 10.0$	88.73	87.52	88.2	87.16	88.72	87.2
$\sigma_B = 12.5$	73.24	73.4	74.9	73.22	74.66	73.75
$\sigma_B = 15.0$	60.9	59.42	61.18	59.28	61.68	60.96
$\sigma_B = 17.5$	48.6	47.64	49.28	49.88	50.26	48.54
$\sigma_B = 20.0$	46.8	39.46	40.62	40.96	40.74	38.6

RESULTS OF DIFFERING INCORRECTLY SPECIFIED CORRELATION STRUCTURES

Vector regression has a lot of features with the longitudinal studies where they exhibit the vectors correlation and independence across the vectors. The primary interests of the regression problems are to estimate and carry the out joint inferences. A vital feature of the longitudinal setting is that the variances and correlations need not be right all the time. Data points can be missing, but it doesn't affect the power significantly as we are carrying out simulations of about 5000 simulations. The different kinds of correlation structures used are mentioned below.

Table 18: Results of T5 Analysis, Gradually Increasing difference between groups

Gradual Increasing difference between groups	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
AR(1)	99.88	99.84	99.88	99.8	99.92	99.9
CS	99.88	99.84	99.88	99.8	99.92	99.9
Toep	99.88	99.84	99.88	99.8	99.92	99.9
Weighted	99.88	99.84	99.88	99.8	99.92	99.9
VC	99.88	99.84	99.88	99.8	99.92	99.9
UN	99.88	99.84	99.88	99.8	99.92	99.9

Table 19: Results of T8 Analysis, Gradually Increasing difference between groups that levels out

Gradual Increasing difference between groups that levels out	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
AR(1)	99.9	99.82	99.86	99.82	99.94	99.88
CS	99.9	99.82	99.86	99.82	99.94	99.88
Toep	99.9	99.82	99.86	99.82	99.94	99.88
Weighted	99.9	99.82	99.86	99.82	99.94	99.88
VC	99.9	99.82	99.86	99.82	99.94	99.88
UN	99.9	99.82	99.86	99.82	99.94	99.88

The concept of this is to consistently model the vectors which are evaluated using the power analysis of the model, and It also helps us in understanding the errors in the model. The next session, we will be looking at the different variability of the groups.[11]

RESULTS OF DIFFERENT VARIABILITY IN GROUPS

In the repeated measures analysis using the GLS, we can partition the subject variability and the variability in the error terms. The Repeated measures ANOVA uses the model, which includes zero or more independent variables. In this section, we will be using the dependent sample T-Test because it can also be compared to the mean scores. The repeated measures analysis is also called as the analysis of the dependencies[13].

Table 20: Results of T5 Analysis, Gradually Increasing difference between groups

Increasing Variability in both group $\sigma_A = \sigma_B$	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
5,6,7,8,9	85.74	78.1	85.32	77.96	87.1	77.92

Table 21: Results of T8 Analysis, Gradually Increasing difference between groups and levels out

Increasing Variability in one group σ_B	Autoregressive 0.25		Autoregressive 0.5		Autoregressive 0.75	
	P1	P2	P1	P2	P1	P3
5,6,7,8,9	95.66	92.36	95.64	92.04	95.66	92.7

V. SUMMARY AND FINAL THOUGHTS

The First Method of our analysis was to have SAS vs R comparison to replicate the dental data results, and we were able to AIC is calculated as $-2LL + 2d$ in SAS with LL being the maximum value of the log-likelihood and d the dimension of the model. In the case of local likelihood estimation, d represents the significant number of estimated covariance parameters. In this case, that is two parameters as shown in your output.[5]

On the other hand, R uses the degrees of freedom as calculated by Pinheiro and Bates. And they have a vastly different interpretation of degrees of space in the context of a mixed model as the one used by SAS.[4] AIC and BIC are both penalized-likelihood criteria. They are used for choosing the best predictor subsets in regression and often used for comparing non-nested models, which standard statistical tests cannot do.

The AIC or BIC for a model is usually written in the form $[-2\log L + kp]$, where L is the likelihood function, p is the number of parameters in the model, and k is 2 for AIC and $\log(n)$ for BIC.[5]

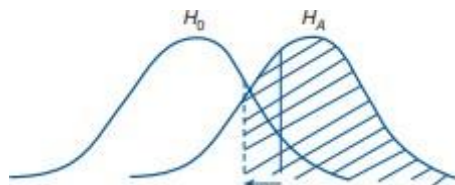
AIC is an estimate of a constant plus the relative distance between the unknown proper likelihood function of the data and the fitted likelihood function of the model so that a lower AIC means a model is considered to be closer to the truth. BIC is an estimate of a process of the posterior probability of a model being real, under a particular Bayesian setup, so that a lower BIC means that a model is considered to be more likely to be the actual model. Both criteria are based on various assumptions and asymptotic approximations. Each, despite its heuristic usefulness, has therefore been criticized as having questionable validity for real-world data. But despite various subtle theoretical differences, their only difference in practice is the size of the penalty; BIC penalizes model complexity more heavily. The only way they should disagree is when AIC chooses a larger model than BIC.[6]

We created a baseline study to have the data simulated, and we have concluded with 5000 simulations created for the autoregressive structure of 0.25, 0.50, 0.75. These are the factors which affect the power. We have considered differences in characteristics like Variance, Correlation, Standard Deviation, Mean, Number of Repeated measure groups, and the different number of time points. This project was used to determine the impact on the power of formal comparisons between groups based on the final measurement when intermediate measurements are included in the analysis via a linear mixed model.

This impact was assessed for varying numbers of intermediate measurements. The nature and extent of this impact will be evaluated for at least some of the following conditions and possibly others as the research develops in the discussion chapter.

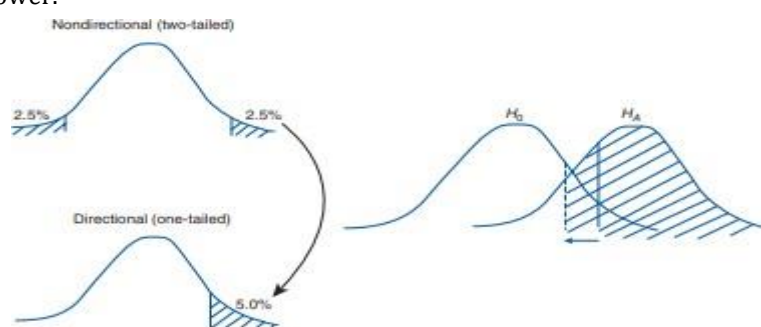
To reject the null hypothesis, we look at the big difference between the groups. We have two kinds of errors which is Type I error and Type II error. Experimental studies need to correctly detect when there is a real difference in the groups and also when there is no significant difference in the groups, which are the error. When the difference between the subjects is not substantial enough to reject the null hypothesis, we either retain the null hypothesis correctly, or we have a Type I error.[8]

We have the simulations increased from 1000 to 5000 to have the comparison more diverse. We also found out that we have statistically significant results with 5000 simulations. We used the GLS function to calculate the repeated measure analysis and performed contrasts to get the p-value which undergoes the power analysis significant tests.

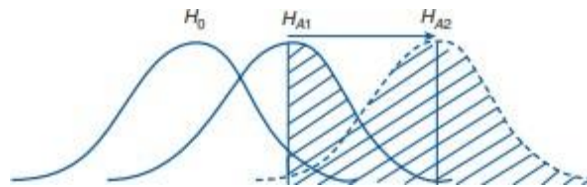


As the Type 1 error increases the power will also increase.

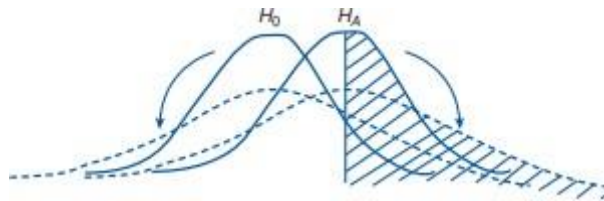
We have the directional and the non-directional power hypothesis where if the Type 1 Error increases, the direction increases power.



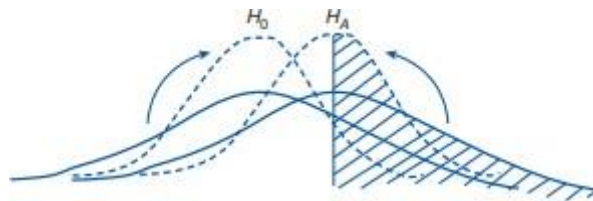
This directionality refers to the variance change in power analysis—the Non Directional hypothesis where the experimental group has higher power than the usual subjects. We can see that as the variability increases the power decreases. Our sample data have more significant variability. Random sampling error is more likely to produce considerable differences between the experimental groups even when there is no real effect. If the sample data in Study A have sufficient variability, a random error might be responsible for the massive difference.



As the difference between the mean increases, we have the power which is increasing—having a baseline simulation study of T5, which is the gradually increasing and T8 which is growing progressively but levels out.



No matter how the samples are simulated, the action difference between the effect size increases and the power also increases.



The higher the error of the variance, the less the power and the graph above can be used to under this principle. The increase in the variability within the groups will decrease the ability to find the difference that does exist. When the data is analyzed within the group, the error variance will mask the effect between the subjects. Anything that makes a difference in the variability, which is decreasing within the groups, there will be an increase in the power as the variability decreases.

The greater the sample size, the higher than the power. This is the principle which is used in the previous graph. We also learned that the preceding section that anything which increases the value of the denominator, the power decreases. Anything that decreases the ability, where the denominator of the test statistic, larger the sample size, we will have more data to analyze.

VI. CONCLUSION

In our comprehensive analysis, we compared SAS and R in replicating dental data results, focusing on AIC and BIC calculations, revealing subtle theoretical differences but a practical divergence only in the size of the penalty. Through 5000 simulations, we examined the autoregressive structure and factors such as Variance, Correlation, Standard Deviation, Mean, and Number of Repeated measure groups, assessing the impact on the power of formal comparisons between groups in a linear mixed model. Our findings highlight that as Type 1 error and the difference between the mean increase, so does the power, while an increase in variability within groups decreases it. The directionality in power analysis was also observed, showing that as variability increases, power decreases, particularly in our sample data with significant variability. The study also emphasized that the greater the sample size, the higher the power, a principle reflected in our graphs. This project has provided valuable insights into the complex interplay of statistical factors in mixed models, contributing to a deeper understanding of error types, effect sizes, and the influence of sample size and variability on statistical power.

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